The decays $\bar{B} \to \bar{K} D$ and $\bar{B} \to \bar{K} \bar{D}$ and final state interactions

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Abstract

The decays $\bar{B} \to \bar{K} D$ and $\bar{B} \to \bar{K} \bar{D}$ taking into account final state interactions are discussed. These decays are described by four strong phases $\delta_0, \delta_1, \tilde{\delta}_0, \tilde{\delta}_1$(subscript 0 and 1 refer to $I = 0$ and $I = 1$ isospin final states), one weak phase γ and four real amplitudes. Isospin constraints are taken into account. It is argued that strong interaction dynamics gives $\delta_1 \approx \tilde{\delta}_1$. The four real amplitudes are estimated. Some observable consequences are discussed.

The weak decays $\bar{B} \to \bar{K}D$ and $\bar{B} \to \bar{K}\bar{D}$ taking into account final state interactions have been studied by several authors [1,2,3,4]. In this paper we elaborate some of the points discussed in reference [4]

These decays are described by four real amplitudes, four strong phases and one weak phase γ. Since the effective weak Hamiltonian for these decays has $\triangle I = 1/2$, the isospin analysis give [4].

$$A(\bar{B} \to K^- D^0) = 2f_1 e^{i\delta_1} \tag{1a}$$

$$A(\bar{B}^0 \to K^- D^+) = [f_1 e^{i\delta_1} + f_0\ e^{i\delta_0}] \tag{1b}$$

$$A(\bar{B}^0 \to \bar{K}^0 D^0) = [f_1 e^{i\delta_1} - f_0\ e^{i\delta_0}] \tag{1c}$$

where δ_0 and $\delta_1$are the phase shifts for $I = 0$ and $I = 1$ isospin states. On other hand for the decays $\bar{B} \to \bar{K}\bar{D}$, we have

$$A(\bar{B}^0 \to \bar{K}^0 \bar{D}^0) = 2\tilde{f}_1\ e^{i\gamma}\ e^{i\tilde{\delta}_1} \tag{2a}$$

$$A(B^- \to K^- \bar{D}^0) = e^{i\gamma}\ [\tilde{f}_1\ e^{i\tilde{\delta}_1} + \tilde{f}_0\ e^{i\tilde{\delta}_0}] \tag{2b}$$

$$A(B^- \to \bar{K}^0 D^-) = e^{i\gamma}\ [-\tilde{f}_1\ e^{i\tilde{\delta}_1} + \tilde{f}_0\ e^{i\tilde{\delta}_0}] \tag{2c}$$

¿From Eqs. (1) and (2), we obtain

$$R \equiv \frac{\Gamma(\bar{B}^0 \to K^- D^+) - \Gamma(\bar{B}^0 \to \bar{K}^0 D^0)}{\Gamma(\bar{B}^0 \to K^- D^+) + \Gamma(\bar{B}^0 \to \bar{K}^0 D^0)} = \frac{2f_1 f_0 \cos(\delta_1 - \delta_0)}{f_1^2 + f_0^2} \tag{3}$$

$$\tilde{R} \equiv \frac{\Gamma(B^- \to K^- \bar{D}^0) - \Gamma(B^- \to \bar{K}^0 D^-)}{\Gamma(B^- \to \bar{K}^0 D^-) + \Gamma(\bar{B} \to \bar{K}^0 D^-)} = \frac{2\tilde{f}_1 \tilde{f}_0 \cos(\tilde{\delta}_1 - \tilde{\delta}_0)}{\tilde{f}_1^2 + \tilde{f}_0^2} \tag{4}$$

Further if we consider the decay of $B^{\mp}$ to CP-eigenstates $D_{1,2} = (D^0 \mp \bar{D}^0)/\sqrt{2}$ (in our convention D_1 and D_2 have $CP = +1$ and -1 respectively), we obtain

$$R_{1,2} \equiv \Gamma(B^- \to K^- D_{1,2}) + \Gamma(B^+ \to K^+ D_{1,2})/\Gamma(B^- \to K^- D^0)$$

$$= [1 + \frac{1}{4}(r_1^2 + r_0^2 + 2r_1 r_2 \cos(\tilde{\delta}_1 - \tilde{\delta}_0) \mp r_1 \cos\gamma \cos(\tilde{\delta}_1 - \delta_1) \mp r_0 \cos\gamma \cos(\bar{\delta}_1 - \delta_1)] \tag{5}$$

$$
\begin{aligned}
\mathcal{A}_{1,2} &\equiv \frac{\Gamma(B^- \to K^- D_{1,2}) - \Gamma(B^+ \to K^+ D_{1,2})}{\Gamma(B^- \to K^- D^0)} \\
&= \pm[r_1 \sin\gamma(\tilde{\delta}_1 - \delta_1) + r_0 \sin\gamma \sin(\tilde{\delta}_0 - \delta_1)]
\end{aligned} \tag{6}
$$

where

$$
r_1 = \tilde{f}_1/f_1, r_0 = \tilde{f}_0/f_1 \tag{7}
$$

It may be noted that we get the result of reference [4] if we put $\tilde{f}_0 = \tilde{f}_1$ and $\tilde{\delta}_0 = \tilde{\delta}_1$.

So far our analysis is general. To proceed further we note that these decays are determined by the tree amplitude T, the color suppressed amplitudes $C(\tilde{C})$ and annihilation amplitude $\tilde{A}$. In terms of these amplitudes

$$
f_1 = \frac{G_F}{\sqrt{2}}|V_{cb}V_{us}^*|\frac{1}{2}(T + C) \tag{8a}
$$

$$
f_0 = \frac{G_F}{\sqrt{2}}|V_{cb}V_{us}^*|\frac{1}{2}(T - C) \tag{8b}
$$

and

$$
\tilde{f}_1 = \frac{G_F}{\sqrt{2}}|V_{ub}V_{cs}^*|\frac{1}{2}\tilde{C} \tag{9a}
$$

$$
\tilde{f}_0 = \frac{G_F}{\sqrt{2}}|V_{ub}V_{cs}^*|\frac{1}{2}(\tilde{C} + 2\tilde{A}) \tag{9b}
$$

In the Wolfenstein representation of CKM matrix [5]

$$
\left|\frac{V_{ub}V_{cs}}{V_{cb}V_{us}^*}\right| = \sqrt{\rho^2 + \eta^2} \tag{10}
$$

Thus we get

$$
r_1 = \sqrt{\rho^2 + \eta^2}\left(\frac{\tilde{C}}{T + C}\right) \tag{11}
$$

$$
r_0 = \sqrt{\rho^2 + \eta^2}\left(\frac{\tilde{C} + 2\tilde{A}}{T + C}\right) \tag{12}
$$

$$
R = \frac{T^2 - C^2}{T^2 + C^2}\cos(\delta_1 - \delta_0) \simeq (1 - 2\frac{C^2}{T^2})\cos(\delta_1 - \delta_0) \tag{13}
$$

$$\tilde{R} = \frac{2\tilde{C}(\tilde{C}+2\tilde{A})}{\tilde{C}^2+(\tilde{C}+2\tilde{A})^2}\cos(\tilde{\delta}_1-\tilde{\delta}_0) \approx (1-2\tilde{A}^2/\tilde{C}^2)\cos(\tilde{\delta}_1-\tilde{\delta}_0) \tag{14}$$

where we have retained only the terms upto C^2/T^2 and $\tilde{A}^2/\tilde{C}^2$, since C^2/T^2 and $\tilde{A}^2/\tilde{C}^2$ are small (see below).

The following remarks about the strong phases are in order. Consider the S-wave scattering

$$\bar{K} + D \rightarrow \bar{K} + D \tag{15}$$

$$\bar{K} + \bar{D} \rightarrow \bar{K} + \bar{D} \tag{16}$$

Since $\bar{K} \sim s\bar{q}$ and $D \sim c\bar{q}$, $q = u$ or d,no s and u channels poles are allowed, whereas since $\bar{D} \sim q\bar{c}$, the poles with the quantum number of D^-_{so} are possible in these channels. But these states carry $I = 0$, hence these states will contribute to $I = 0$ scattering amplitude i.e. to $\bar{\delta}_0$. The t channel is common to the processes (15) and (16) and the lowest lying poles which can contribute are ρ and σ. The ρ and $\sigma-$ pole, contribute to $I = 1$ and $I = 0$ channel, respectively. Thus it is reasonable to assume that $(\delta_1 = \tilde{\delta}_1)$; since s and u-channels poles do not contribute to $I = 1$ scattering amplitudes. Thus with $\delta_1 = \tilde{\delta}_1$, we obtain from Eqs. (5) and (6)

$$R_2 - R_1 = 2\cos\gamma\left(r_1 + r_0\cos(\tilde{\delta}_0-\tilde{\delta}_1)\right) \tag{17}$$

$$\mathcal{A}_{1,2} = \pm\, r_0 \sin\gamma \sin(\tilde{\delta}_0-\tilde{\delta}_1) \tag{18}$$

First we note that if $2(\tilde{A}/_{\tilde{C}}) << 1$, then Eq. (14) gives cos $(\tilde{\delta}_0-\tilde{\delta}_1)$ in term of $\tilde{R}$, and then from Eq. (18) one can extract $r_0\sin\gamma$. If $r_1 \simeq r_0$ which is the case if $2(\tilde{A}/_{\tilde{C}}) << 1$, then, Eqs. (14), (17) and (18) give us information about r_0 and the weak phase γ.

Before we give some estimates for the amplitudes $T, C(\tilde{C})$ and $\tilde{A}$, we discuss $SU(3)$ relations for the various amplitudes for the decays of $\bar{B}$ described by the effective Lagrangians:

$$L_{eff} = \frac{G_F}{\sqrt{2}}V_{cb}V^*_{us}[\bar{s}\gamma_\mu(1+\gamma_5)u][\bar{c}\gamma_\mu(1+\gamma_5)b] \tag{19}$$

$$L_{eff}=\frac{G_F}{\sqrt{2}}V_{ub}V_{cs}^{*}[\bar{s}\gamma_\mu(1+\gamma_5)c][\bar{u}\gamma_\mu(1+\gamma_5)b] \tag{20}$$

$SU(3)$ analysis of these decays gives the following relations between various amplitudes

$$A(B^-\to K^-D^0)=A(\bar{B}^0\to K^-D^+)+A(\bar{B}^0\to\bar{K}^0D^0) \tag{21a}$$

$$A(\bar{B}_s^0\to\pi^-D^+)=\sqrt{2}A(\bar{B}_s^0\to\pi^0D^0) \tag{21b}$$

$$\sqrt{6}A(\bar{B}_s^0\to\eta D^+)=A(\bar{B}_s^0\to\pi^-D^+)-2A(\bar{B}_s^0\to\bar{K}^0D^0) \tag{21c}$$

and

$$A\left(B^-\to K^-\bar{D}^0\right)=A(B^-\to\bar{K}^0D^-)+A(\bar{B}^0\to\bar{K}^0\bar{D}^0) \tag{22a}$$

$$A\left(\bar{B}^0\to\pi^+D_s^-\right)=\sqrt{2}A\left(B^-\to\pi^0D_s^-\right) \tag{22b}$$

$$A\left(\bar{B}_s^0\to\pi^+D^-\right)=\sqrt{2}A\left(\bar{B}_s^0\to\pi^0D^0\right) \tag{22c}$$

$$\sqrt{6}A\left(B^-\to\eta D_s^-\right)=A(\bar{B}^0\to\pi^+D_s^-)-2A(B^-\to\bar{K}^0D^-) \tag{22d}$$

$$\sqrt{6}A\left(\bar{B}_s^0\to\eta\bar{D}^0\right)=A(\bar{B}_s^0\to\pi^+D^-)-2A(\bar{B}^0\to\bar{K}^0\bar{D}^0) \tag{22e}$$

It may be noted that relations $(21,a,b)$ and $(22a,b,c)$ also follows from isospin analysis only.

We now calculate the amplitudes, $T, C(\tilde{C})$ and $\tilde{A}$ from the effective Lagrangians (19) and (20). In the factorization anstz, they are given by

$$T=a_1f_KF_0^{B-D}(m_K^2)(m_B^2-m_D^2) \tag{23}$$

$$C=a_2f_DF_0^{B-K}(m_D^2)(m_B^2-m_K^2)=\tilde{C} \tag{24}$$

$$\tilde{A}=a_1f_BF_0^{D-K}(m_B^2)(m_D^2-m_K^2) \tag{25}$$

where the form factor F_0 is defined as $[t=(p-p')^2]$ $(q=c$ or $u)$

$$\begin{aligned}&\langle P(p')\,|i\bar{q}\gamma_\mu(1+\gamma_5)b|\,B(p)\rangle\\ &\sim[F_+(t)(p+p')_\mu+F_-(t)(p-p')_\mu]\\ &=\left[(p+p')_\mu-\frac{m_B^2-m_P^2}{t}(p+p')_\mu\right]F_1(t)\\ &\quad+\left[\frac{m_B^2-m_P^2}{t}(p-p')_\mu\right]F_0(t)\end{aligned} \tag{26}$$

One can get some information for the form factor $F_0^{B-D}(t)$ from the heavy quark effective theory [6], but for the form factors involving one light meson we use a model which is based on dispersion relations, using once-subtracted dispersion relation for $[F_+(t) + F_-(t)]$ and unsubtracted dispersion relatiuon for $[F_t(t) + F_-(t)]$ as given in reference [7]. Retaining only the contribution from the low lying states $B^*(1^-)$ and $B_0(O^+)$ in the dispersion relations, one gets

$$F_1(t) = \frac{1}{2}\left[\frac{f_B}{f_P} - \frac{f_{B^*}g_{B^*BP}}{m_{B^*}} - \frac{f_{B_0}g_{B_0BP}}{m_{B_0}} + \frac{2f_{B^*}g_{B^*BP}}{m_{B^*}^2 - t}\right] \tag{27a}$$

$$\begin{aligned} F_0(t) = {} & \frac{1}{2}\left\{\left[\frac{f_B}{f_P} - \frac{f_{B^*}g_{B^*BP}}{m_{B^*}} - 2f_{B_0}g_{B_0BP}\frac{m_{B_0}}{m_B^2} + \frac{f_{B_0}g_{B_0BP}}{m_{B_0}}\right]\right. \\ & + \frac{t}{m_B^2}\left[\frac{f_B}{f_P} - \frac{f_{B^*}g_{B^*BP}}{m_{B^*}}\right] \\ & \left. +2\left[\frac{m_{B_0}}{m_B^2}f_{B_0}g_{B_0BP}\frac{m_{B_0}^2 - m_B^2}{m_{B_0}^2 - t}\right]\right\} \end{aligned} \tag{27b}$$

If we demand that there should not be a term depending linearly on t in $F_0(t)$, we get the sum rule [8]

$$\frac{f_B}{f_P} = \frac{f_{B^*}g_{B^*BP}}{m_{B^*}} + \frac{f_{B_0}g_{B_0BP}}{m_{B_0}} \tag{28}$$

On using Eq. (28), we obtain from Eqs. (27), simple expressions for $F_1(t)$ and $F_0(t)$:

$$F_1(t) = \frac{f_{B^*}m_{B^*}g_{B^*BP}}{m_{B^*}^2 - t} \tag{29}$$

$$F_0(t) = \left[\frac{f_B}{f_P} - \frac{m_{B_0}^2}{m_{B_0}^2}(1 - \frac{m_{B_0}^2 - m_B^2}{m_{B_0}^2 - t})(\frac{f_B}{f_P} - \frac{f_{B^*}g_{B^*BP}}{m_{B^*}})\right] \tag{30}$$

Paramaterising

$$g_{B^*BP} = \lambda_B\frac{m_{B^*}}{f_P} \tag{31}$$

and using the relation [9]

$$f_{B^*} = f_B \tag{32}$$

we get

$$F_1(t) = \lambda_B \frac{f_B}{f_P} \frac{m_{B^*}^2}{m_{B^*}^2 - t} \tag{33}$$

$$F_0(t) = \frac{f_B}{f_P} \left\{ \lambda_B - \frac{m_{B_0}^2 - m_B^2}{m_B^2} (1 - \lambda_B) \left[1 - \frac{m_{B_0}^2}{m_{B_0}^2 - t} \right] \right\} \tag{34}$$

$$F_1(0) = F_0(0) = \lambda_B \frac{f_B}{f_P} \tag{35}$$

These form factors except for λ_B depend upon the ratio $\frac{f_B}{f_P}$ and masses m_B, m_{B^*} and m_{B_0} which can be extracted from the experimental data. We assume that λ_B and λ_D scale as:

$$\lambda_B = \frac{\Lambda}{m_B}, \lambda_D = \Lambda / m_D \tag{36}$$

where Λ is a scale characteristic of bound state which we take 1 GeV. Using this assumption, we get

$$\begin{aligned} \Gamma\left(D^* \to D^0 \pi^+\right) &= \frac{g_{D^* D\pi}^2}{6\pi} \frac{p_\pi^3}{m_{D^*}^2} \\ &= \frac{1}{6\pi} \left(\frac{\Lambda}{m_D} \frac{m_{D^*}}{f_\pi} \right)^2 \frac{p_\pi^3}{m_{D^*}^2} \\ &= \frac{1}{6\pi} \left(\frac{\Lambda}{m_D} \right)^2 \frac{p_\pi^3}{f_\pi^2} \approx 52 KeV \end{aligned} \tag{37}$$

Thus we obtain

$$\Gamma(D^{**} \to D\pi) = \Gamma(D^{*+} \to D^0 \pi^+) + \Gamma(D^{*+} \to D^+ \pi^0) \simeq 78 \text{ KeV} \tag{38}$$

to be compared with the experimental upper limit [10] $\Gamma < 113$ KeV. Thus our assumption that $\lambda_D = \Lambda / m_D$ can be tested experimentally. Finally using Eqs. (36), we get

$$F_0^{D-K}(m_B^2) = \frac{f_B}{f_K} \left[\frac{\Lambda}{m_D} - \frac{m_{B_0}^2 - m_B^2}{m_B^2} (1 - \frac{\Lambda}{m_D})(1 - \frac{m_{B_0}^2}{m_{B_0}^2 - m_D^2}) \right] \tag{39}$$

$$F_0^{D-K}(m_B^2) = \frac{f_{D_s}}{f_K} \left[\frac{\Lambda}{m_{D_s}} - \frac{m_{s_0}^2 - m_D^2}{m_D^2} (1 - \frac{\Lambda}{m_{D_s}})(1 - \frac{m_{D_{s_0}}^2}{m_{D_{s_0}}^2 - m_B^2}) \right] \tag{40}$$

Using following values for the masses (in GeV): [11] $m_B = 5.279, m_{B_0} = 5.60, m_{D_s} = 1.968, m_{D_{s_0}} = 2.357, m_D = 1.869$ and $f_D = 200 MeV, f_{D_s} = 240 MeV, f_B = 180 MeV$ and $f_K = 158 MeV$[6], we obtain

$$F_0^{B-K}(m_B^2) \simeq 0.202\, \frac{f_B}{f_K} \simeq 0.23 \tag{41}$$

$$F_0^{D-K}(m_B^2) \simeq 0.145\, \frac{f_{D_S}}{f_K} \simeq 0.22 \tag{42}$$

Hence we get

$$\tilde{A}/\tilde{C} = \frac{a_1}{a_2}\, \frac{f_B}{f_K}\, \frac{F_0^{D-K}(m_B^2)}{F_0^{B-K}(m_D^2)} \left(\frac{m_D^2 - m_K^2}{m_B^2 - m_K^2} \right) \simeq 0.120 \tag{43}$$

$$C/T = \frac{a_2}{a_1}\, \frac{f_D F_0^{B-K}(m_D^2)(m_B^2 - m_K^2)}{f_K F_0^{D-K}(m_K^2)(m_B^2 - m_D^2} \simeq 0.126 \tag{44}$$

where we have used for the color suppression factor [12]

$$\frac{a_2}{a_1} = 0.26 \tag{45}$$

and [6]

$$F_0^{B-D}(m_K^2) = 0.587 \tag{46}$$

Now using Eqs. (43), and (44), and $\sqrt{\rho^2 + \eta^2} \simeq 0.36$[13] we get from Eqs. (11), (12), (13), (14), (17) and (18)

$$r_1 \approx 0.040, r_0 \approx 0.050, r_0/r_1 = 1.25 \tag{47}$$

$$R \approx 0.968 \cos(\delta_1 - \delta_0) \tag{48}$$

$$\tilde{R} \approx 0.971 \cos(\tilde{\delta}_1 - \tilde{\delta}_0) \tag{49}$$

$$R_2 - R_1 \approx 0.8 \cos\gamma [1.25 + \cos(\tilde{\delta}_1 - \tilde{\delta}_0)] \tag{50}$$

$$\mathcal{A}_{1,2} \approx \pm 0.05 \sin\gamma \sin(\tilde{\delta}_1 - \tilde{\delta}_0) \tag{51}$$

If the neglect the term $2(\tilde{A}/\tilde{C})^2$, then we have

$$\cos(\tilde{\delta}_1 - \tilde{\delta}_0) \approx \tilde{R} \tag{52}$$

$$\mathcal{A}_{1,2} \approx \pm(0.05)\sqrt{1 - \tilde{R}^2}\, \sin\gamma \tag{53}$$

$$R_2 - R_1 \approx 0.08 \cos\gamma [1.25 + \tilde{R}] \tag{54}$$

To conclude if $2(\tilde{A}/\tilde{C})^2$ is negligible then the branching ratio $\tilde{R}$, the asymmetry $\mathcal{A}_{1,2}$ and $R_2 - R_1$ can give information about the weak phase γ. Conversely if weak phase is known from some other processes, $Eqs.(53), (54)$ and $\tilde{R}$ give us information about r$_0$ and r_0/r_1.

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